Applying the boundary conditions in the first and second equations of (12), one obtains

$$A_1 = -\frac{Qa^2}{G_0\alpha}\frac{J_2(\alpha a)e^{(n_2-n_1)b}}{n_1 e^{(n_2-n_1)b} - n_2} \tag{14}$$

and

$$B_1 = -A_1 e^{(n_1-n_2)b}$$

Then the displacement $(u_\theta)_{z=0}$ is given by

$$(u_\theta)_{z=0} = \frac{Qa^2}{G_0}\int_0^\infty \left[\frac{e^{(n_2-n_1)b} - 1}{n_1 e^{(n_2-n_1)b} - n_2}\right] J_2(\alpha a)J_1(\alpha r)d\alpha \tag{15}$$

To get an idea of how the displacement u_θ changes on $z = 0$ for different values of r, take $m = b = 1$ and $a = k_0 = 0.5$. The result is given in Table 1.

Similar Solutions in Boundary Layer Slip Flow

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Some methods of solution for the boundary layer slip-flow problem have been suggested (see, e.g., Refs. 1–3). To check the accuracy of these solutions, one needs to compare them with exact solutions. The purpose of this note is to show how an exact solution can be obtained by means of the similar-solutions technique.

The basic equation of the incompressible boundary layer in terms of Von Mises' nondimensional coordinates can be written as

$$uu_x = U_e U_{ex} + u(uu_y)_y \tag{1}$$

where U_e is the outer velocity, with the following boundary conditions:

$$u_y(x,0) = k \tag{2}$$

$$u(x,\infty) = U_e \tag{3}$$

$$u(0,y) = g(y) \tag{4}$$

Now put $u^2 = f(x)\,F(z)$, with $z = y/h(x)$.

The functions $f(x)$ and $h(x)$ must be found from Eqs. (1) and (2); one obtains

$$f^{1/2} = h = U_e = 1 + Cx$$

To obtain the function $F(z)$, one needs to solve the following equation:

$$F - (zF'/2) = 1 + (F^{1/2}F''/2C) \tag{5}$$

with the boundary conditions

$$F(\infty) = 1 \qquad F'(0) = 2kF^{1/2}(0)$$

Observe that the function $g(y)$ of Eq. (4) is given by $F(y)$. Equation (5) has been solved numerically, and the results are shown in Fig. 1 for $C = 0.5$ and $k = 1$. Now a satisfactory approximate solution is given which is obtained by substituting for $F^{1/2}$ in Eq. (5) a mean value $F_m^{1/2}(0 < F_m < 1)$. One has then

$$F = 1 + A(1 + 2\zeta)(1 - (\pi\zeta)^{-1/2}\exp(-\zeta) \times \{{}_1F_1(1,\tfrac{1}{2}\cdot\zeta) - [1/(1 + 2\zeta)]\})$$

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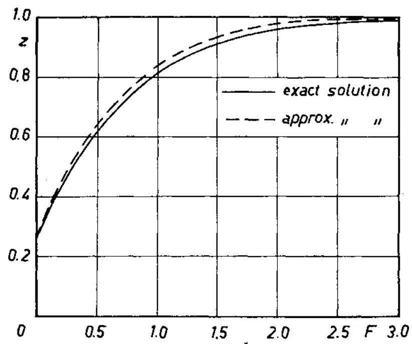

Fig. 1 Exact and approximate velocity function

where ${}_1F_1$ is the confluent hypergeometric function, $\zeta = Cz^2/2F_m^{1/2}$, and the constant A is given by

$$A = \frac{\pi k^2 F_m^{1/2}}{4C} - \frac{\pi k}{4}\left(\frac{k^2F_m}{C^2} + \frac{8F_m^{1/2}}{\pi C}\right)^{1/2}$$

In Fig. 1 is shown this approximate function obtained by assuming for F_m the value 0.5.

References

[1] Hasimoto, H., "Boundary layer slip solutions for a flat plate," J. Aerospace Sci. **25**, 68–69 (1958).

[2] Hassan, H. A., "On skin friction in the slip-flow regime," J. Aerospace Sci. **28**, 335–336 (1961).

[3] Pozzi, A. and Renno, P., "Slip flow with axial pressure gradient," J. Aerospace Sci. **29**, 1393 (1962).

Stress Concentrations around a Small Rigid Spheroidal Inclusion on the Axis of a Transversely Isotropic Cylinder under Torsion

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Introduction

In this note, stresses due to a small rigid inclusion in the form of an oblate spheroid situated on the axis of a large transversely isotropic cylinder under torsion have been found. A corresponding problem for a spherical inclusion in a similar medium was considered by Chatterji.[2] From the results obtained here, stresses due to a rigid inclusion in the form of a prolate spheroid can be deduced by suitable modification.

Solution

The strain-energy function of a transversely isotropic material in cylindrical coordinates is given by

$$W = \tfrac{1}{2}c_{11}(e_{rr}^2 + e_{\theta\theta}^2) + \tfrac{1}{2}c_{33}e_{zz}^2 + c_{13}(e_{rr} + e_{\theta\theta})e_{zz} + c_{12}e_{rr}e_{\theta\theta} + \tfrac{1}{2}c_{44}(e_{\theta z}^2 + e_{rz}^2) + \tfrac{1}{2}c_{66}e_{r\theta}^2$$

where

$$c_{12} = c_{11} - 2c_{66}$$

Considering the large twisted cylinder under torsional stresses

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Table 1 Variation of stress concentration

η =	0	$\pi/6$	$\pi/4$	$\pi/3$	$\pi/2$	$\frac{2}{3}\pi$	$\frac{3}{4}\pi$	$\frac{5}{6}\pi$	π
$2p_{\xi\eta}/P\mu p_c kc$ =	0	0.34	0.36	0.30	0	−0.30	−0.36	−0.34	0

prevailing at infinity, one assumes

$$u_r = 0 \qquad u_\theta = \vartheta = \vartheta(r,z) \qquad u_z = 0 \tag{1}$$

The strain elements then are given by

$$\begin{gathered} e_{rr} = e_{\theta\theta} = e_{zz} = e_{rz} = 0 \\ e_{r\theta} = r(\partial/\partial r)(\vartheta/r) \qquad e_{\theta z} = r(\partial/\partial z)(\vartheta/r) \end{gathered} \tag{2}$$

From the preceding strain components, the stresses can be derived as follows:

$$\begin{gathered} p_{rr} = p_{\theta\theta} = p_{zz} = p_{rz} = 0 \\ p_{r\theta} = c_{66}r(\partial/\partial r)(\vartheta/r) \qquad p_{\theta z} = c_{44}r(\partial/\partial z)(\vartheta/r) \end{gathered} \tag{3}$$

Two of the body stress equations of equilibrium are seen to be satisfied identically, and the third one becomes

$$(\partial p_{r\theta}/\partial r) + (\partial p_{\theta z}/\partial z) + 2(p_{r\theta}/r) = 0$$

Substituting the values of the stress components from Eq. (3), one gets

$$(\partial^2\vartheta/\partial r^2) + (1/r)(\partial\vartheta/\partial r) - (\vartheta/r^2) + k^2(\partial^2\vartheta/\partial z^2) = 0 \tag{4}$$

where $k^2 = c_{44}/c_{66}$. The inclusion is an oblate spheroid whose boundary is given by

$$(r^2/a^2) + (z^2/b^2) = 1 \qquad a > b \tag{5}$$

The inclusion is supposed to be fixed rigidly. Put $z = kz'$ in Eq. (4), which then reduces to

$$(\partial^2\vartheta/\partial r^2) + (1/r)(\partial\vartheta/\partial r) - (\vartheta/r^2) + (\partial^2\vartheta/\partial z'^2) = 0 \tag{6}$$

The boundary becomes

$$(r^2/a^2) + [(z')^2/(b^2/k^2)] = 1$$

Assuming $k > 1$, it will be found that $a^2 > b^2/k^2$. At a great distance from the inclusion it is supposed that $\vartheta(= \vartheta_1)$ is due to torsion of the circular cylinder only. That is, at a great distance from the inclusion, $\vartheta_1 = p_c rz = p_c krz'$, p_c being the twist.

In the case of an oblate spheroid, introduce the transformation given by

$$r + iz' = c\cosh(\xi + i\eta)$$

so that

$$\begin{aligned} r &= c\cosh\xi\cos\eta \\ z' &= c\sinh\xi\sin\eta \\ 1/h^2 &= c^2(\cosh^2\xi - \cos^2\eta) \end{aligned}$$

If one puts

$$\begin{aligned} a &= c\cosh\alpha & b/k &= c\sinh\alpha \\ c &= [a^2 - (b^2/k^2)]^{1/2} & \tanh\alpha &= b/ka \end{aligned}$$

one has $\xi = \alpha$ when r and z' are connected by (7). Equation (6) now becomes

$$(\partial/\partial\xi)[(\partial\vartheta/\partial\xi) + \vartheta\tanh\xi] + (\partial/\partial\eta)[(\partial\vartheta/\partial\eta) - \vartheta\tan\eta] = 0$$

Take as a solution

$$\vartheta = \{A(d/d\xi)[Q_2(i\sinh\xi)]\}(d/\eta)P_2(\sin\eta)$$

where $P_2(\sin\eta)$ and $Q_2(i\sinh\xi)$ are Legendre functions of the first and second kind, respectively. For large ξ, $\vartheta_1 = p_c kc^2 \sinh\xi\cosh\xi\sin\eta\cos\eta$.

Now, the only boundary condition to be satisfied is $\vartheta + \vartheta_1 = 0$ on $\xi = \alpha$, which gives

$$A = -(p_c kc^2/3)\{\sinh\alpha\cosh\alpha/(d/d\alpha)[Q_2(i\sinh\alpha)]\}$$

Hence the total displacement is given by

$$\begin{aligned} \vartheta_2 &= \vartheta + \vartheta_1 \\ &= \frac{p_c kc^2}{3}\left\{\sinh\xi\cosh\xi - \frac{\sinh\alpha\cosh\alpha}{(d/d\alpha)[Q_2(i\sinh\alpha)]}\frac{d}{d\xi}\times\right. \\ &\qquad \left.[Q_2(i\sinh\xi)]\right\}\frac{d}{d\eta}P_2(\sin\eta) \end{aligned}$$

On $\xi = \alpha$, $p_{\xi\xi} = p_{\eta\eta} = 0$, while

$$[p_{\xi\eta}] = \frac{\mu p_c kc}{6}\frac{(d/d\eta)P_2(\sin\eta)}{(\cosh^2\alpha - \cos^2\eta)^{1/2}}\times$$
$$\left\{\frac{2(d/d\alpha)[Q_2(i\sinh\alpha)]\cosh2\alpha - \sinh2\alpha(d^2/d\alpha^2)[Q_2(i\sinh\alpha)]}{(d/d\alpha)[Q_2(i\sinh\alpha)]}\right\}$$

To have an idea of the variation of stress around the inclusion, assume, for example, $\alpha = 1$. Then

$$p_{\xi\eta} = \frac{P\mu P_c k_c}{2}\frac{\sin\eta\cos\eta}{(2.3811 - \cos^2\eta)^{1/2}}$$

where

$$P = \left\{\frac{2(d/d\alpha)[Q_2(i\sinh\alpha)]\cosh2\alpha - \sinh2\alpha(d^2/d\alpha^2)[Q_2(i\sinh\alpha)]}{(d/d\alpha)[Q_2(i\sinh\alpha)]}\right\}$$

Variation of $p_{\xi\eta}$ on the inclusion for different values of η is shown in Table 1.

References

[1] Love, A. E. H., *A Treatise on the Mathematical Theory of Elasticity* (Dover Publications Inc., New York 1927), pp. 252, 160.

[2] Chatterji, P. P., "Stress concentrations around a small spherical inclusion on the axis of a transversely isotropic circular cylinder under tension," J. Assoc. Appl. Physicists **V**, 10–15 (1958).

Stagnation Point Heat Transfer of a Blunt-Nosed Body in Low-Density Flow

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A SHOCK-boundary-layer matching scheme has been introduced in the study of stagnation point heat transfer characteristics of a blunt-nosed body in hypersonic flow where the boundary layer thickness is of the order of the detachment distance. The velocity components, stress components, temperature, heat flux, pressure, and density are matched on a matching surface, which can be determined uniquely from the analysis. The heat transfer results merge with those developed in Ref. 1 for large Reynolds number and show smooth transition from high Reynolds number to Reynolds number of the order of 50 [R_{es}=

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